Functions Need to Know

Revision Guide

Function Notation

$y = f(x)$ is another way of saying "y is a function of x" (think back to function machines).

You need to be able to substitute values into a function.

Example:

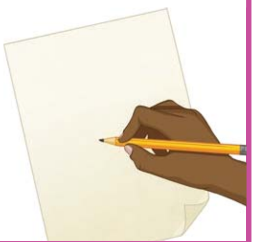
For the function $f(x) = 2x + 5$, evaluate $f(6)$.

You can see the letter x in $f(x)$ has been replaced with the number 6; you need to replace every x in the expression with the number 6.

So $f(6) = 2 \times 6 + 5 = 17$

You try:

For the function $f(x) = 3x - 2$, evaluate $f(11)$.



Answers

You try:

i) Complete the square to find the turning point of $x^2 - 10x + 20$.

$(x - 5)^2 - 25 + 20 = (x - 5)^2 - 5$

The turning point is (5, -5)

ii) Complete the square to find the turning point of $4x^2 + 16x + 9$.

$4(x^2 + 4x) + 9$

$4[(x + 2)^2 - 4] + 9 = 4(x + 2)^2 - 16 + 9 = 4(x + 2)^2 - 7$

The turning point is (-2, -7)

You try:

Describe the transformation which maps the graph $y = f(x)$ to $y = f(x + 4) - 3$.

A translation by the vector $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

You try:

Prove that the sum of two consecutive odd numbers is always a multiple of 4.

Let the smaller of the 2 odd numbers be defined as $2n + 1$. Then, the next odd number is $2n + 3$.

The sum of these is $2n + 1 + 2n + 3 = 4n + 4$

$4n + 4 = 4(n + 4)$

Therefore, this is a multiple of 4.

Answers

You try:

For the function $f(x) = 3x - 2$, evaluate $f(11)$.

$f(11) = 3 \times 11 - 2 = 31$

You try:

Find the inverse $f^{-1}(x)$ of the function $f(x) = 3x - 2$.

$x = 3y - 2$

$x + 2 = 3y$

$\frac{x + 2}{3} = y$

So $f^{-1}(x) = \frac{x + 2}{3}$

You try:

The functions f and g are defined by $f(x) = 5x - 2$ and $g(x) = 3x + 8$. Find $gf(x)$.

$gf(x) = g[f(x)] = 3(5x - 2) + 8$

$gf(x) = 15x - 6 + 8$

$gf(x) = 15x + 2$

Inverse Functions

The inverse of the function $f(x)$ is written as $f^{-1}(x)$. To find the inverse, start by writing your function in terms of x and y, then swap the x and y around. Once you have done this, rearrange to make y the subject.

Example:

Find the inverse of the function $f(x) = 2x + 5$.

Writing the function in terms of x and y gives us

$y = 2x + 5$

Swapping the x and y around gives us

$x = 2y + 5$

Rearranging to make y the subject gives us

$y = \frac{x - 5}{2}$

So, $f^{-1}(x) = \frac{x - 5}{2}$

You try:

Find the inverse $f^{-1}(x)$ of the function $f(x) = 3x - 2$.

Composite Functions

A composite function is the result of applying two or more functions in a row.

Example:

The functions f and g are defined by $f(x) = 3x + 7$ and $g(x) = 4x$. Find $fg(x)$.

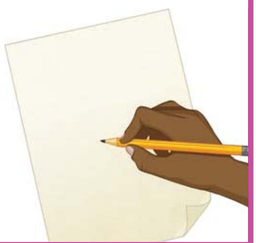
We need to rewrite our $fg(x)$ as $f[g(x)]$. Once we have done that, we can see that, in the function $f(x)$, we need to replace every x with $g(x)$.

So, $f[g(x)] = 3(4x) + 7$

$= 12x + 7$

You try:

The functions f and g are defined by $f(x) = 5x - 2$ and $g(x) = 3x + 8$. Find $gf(x)$.



Notes

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Completing the Square

Sometimes, the exam paper will ask you to complete the square; other times, it will ask you to write an expression in the form $(x + a)^2 + b$ (or similar).

Example:

Write $x^2 + 8x + 10$ in the form $(x + a)^2 + b$, where a and b are constants.

First, we halve the coefficient of x (which is 8) to get $(x + 4)^2$.

Then, we square that number to get $4^2 = 16$, which we now subtract from our brackets before re-adding the 10.

$(x + 4)^2 - 16 + 10 = (x + 4)^2 - 6$

The turning point of this is (-4, -6).

In general, the turning point of a function $f(x) = (x + a)^2 + b$ is $(-a, b)$.

You try:

i) Complete the square to find the turning point of $x^2 - 10x + 20$.

ii) Complete the square to find the turning point of $4x^2 + 16x + 9$.

Remember, if the coefficient of x^2 is larger than 1, you'll need to first factorise the first two terms.

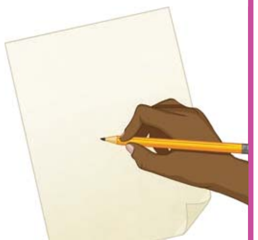
Transforming Graphs

For a function $y = f(x)$, there are four transformations you need to know:

1. $f(x) + a$ is a translation by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$;
2. $f(x + a)$ is a translation by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$;
3. $-f(x)$ is a reflection in the x-axis;
4. $f(-x)$ is a reflection in the y-axis.

You try:

Describe the transformation which maps the graph $y = f(x)$ to $y = f(x + 4) - 3$.



Proof

Remember: to disprove a statement, you just need to provide one example which doesn't work.

Example 1:

Alison says that the sum of two square numbers is always an odd number. Is she right?

No, $4^2 + 6^2 = 52$, which is even.

To prove a statement, you'll need to use algebra. The important expressions are (where n is a positive whole number):

- $2n$ is always an even number;
- $2n + 1$ is always an odd number;
- consecutive means 'in a row'.

Example 2:

Prove that the product of two consecutive even numbers is always an even number.

Let the smaller of the 2 even numbers be defined as $2n$. Then the next even number will be $2n + 2$.

The product is $2n(2n + 2) = 4n^2 + 4n$.

$4n^2 + 4n$ can be written as $2(2n^2 + 2)$, therefore 2 is a factor and the product is always even.

You try:

Prove that the sum of two consecutive odd numbers is always a multiple of 4.